Worksheet 5 solutions

Wednesday, February 17, 2021 9:14 AM

1.
   a. taylor expand f about p_n: $f(x) = f(p_n) + (x - p_n)f'(p_n) + \frac{(x-p_n)^2}{2} f''(\xi)$
   b. plug in $x = p$: $f(p) = f(p_n) + (p - p_n)f'(p_n) + \frac{(p-p_n)^2 f''(\xi)}{2}$
   c. divide by $f'(p_n)$: $\frac{f(p)}{f'(p_n)} = \frac{f(p_n)}{f'(p_n)} + (p - p_n) + (p - p_n)^2 \frac{f''(\xi)}{2f'(p_n)}$
   d. note $f(p) = 0$ and $p_{n+1} = p_n - \frac{f(p_n)}{2f'(p_n)}$
   e. so we get $0 = p - p_{n+1} + (p - p_n)^2 \frac{f''(\xi)}{2f'(p_n)}$
   f. rearrange, take absolute value, and we are done.
2.
   a. let $x = p_1^{(0)}$, then by defintition, we have:
   b. $.75 = 1 - \dfrac{(x-1)^2}{3 - 2x + 1}$
   c. solve for x, get $x = 0$ and $x = 1.5$
3.

$$
\begin{array}{c|ccccc}
1 & 7 & 0 & -2 & -5 & -3 \\
 & & 7 & 7 & 5 & 0 \\
\hline
 & 7 & 7 & 5 & 0 & -3
\end{array}
$$

4. d
   a. $f(x)$ already is the unique polynomial going through these points
5. d

	-1	0	1
f_0	0	-1	0
f_1	-1	0	5
f_2	-3	0	1

   a. then $g = f_0 + f_1 + f_2$